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Natural convection due to a heat source on a vertical plate

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Abstract

In this work, the flow over a discrete heat source flush mounted on a vertical adiabatic surface is studied. The two- and three-dimensional formulations were adopted in order to compare with the results presented in the literature. The flow was assumed incompressible and laminar with constant fluid properties under Boussinesq assumption. The finite volume control method was used for the discretization of the elliptic governing equations. The numerical results presented confirm in part the theoretical behavior and show the existence of a transition between the two- and three-dimensional plume. Additionally, similarity scales for the thermal boundary layer were proposed for the *xy* and *zy* planes. The presence of this region of transition emphasizes the complexity of this kind of flow, where two- or three-dimensional effects must not be studied separately, mainly on electronic packaging where heat sources are in that transition region.

Keywords: Natural convection; Three-dimensional effects

1. Introduction

Natural convection heat transfer is present in many engineering applications, such as solar collectors, environmental engineering and electronic packaging. A large amount of papers were published dealing with natural convection in two-dimensional enclosures, channels and plates, considering the variation of different parameters, such as Rayleigh number, Prandtl number, aspect ratio, radiation, conduction, variable properties and discrete sources. For the cooling of electronic packaging, where power distribution and location of discrete heat sources are very important, natural convection is the only heat transfer mode in case of artificial cooling failure [1–6].

Although two-dimensional models are very useful to the understanding of the physical phenomenon due to their simplicity, they are still limited to explain many complex real situations, where three-dimensional effects become more important to the fluid transport. Some of these effects have been included in the works of Fujii [7] and Jaluria [1], for an axisymmetric plume flow over point sources, and in the paper of Kurdyumov and Liñán [9], to study the flow over hemispherical sources. A more realistic modeling is to consider the interaction between the plume and other surfaces, such as vertical surfaces, as in the works of Carey and Mollendorf [10] and Higuera and Weidman [11].

In this work, the flow over a discrete heat source flush mounted on a vertical adiabatic surface is studied (see Fig. 1). The two- and three-dimensional formulations were adopted in order to compare with the results presented in the literature. In the two-dimensional case, the heat source was a strip with 0.01 m of height and, on the three-dimensional one, it was a 0.01 m square. The adiabatic wall was 1.5 m height and 1.0 m depth, and the heat source was mounted at the centerline in the wall and at 0.15 m from the bottom edge.

2. Analysis

2.1. Problem formulation

The flow was assumed incompressible and laminar with constant fluid properties except for the density

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Nomenclature

d	heat source characteristic length
g	gravitational acceleration
Gr_d^*	modified Grashof number, $g\beta d^4 q''/k\nu^2$
k	thermal conductivity
P	dimensionless pressure, $(p^+/\rho)(d/\alpha)^2$
Pr	Prandtl number, ν/α
q''	heat flux
Ra_d^*	modified Rayleigh number, $Gr_d^* Pr$
T	dimensionless temperature, $(T^+ - T_\infty^+)/(q''d/k)$
u	x dimensionless velocity, $u^+ d/\alpha$
v	y dimensionless velocity, $v^+ d/\alpha$
w	z dimensionless velocity, $w^+ d/\alpha$
x, y, z	spatial coordinates

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
ν	kinematic viscosity
ϕ	surface temperature excess ratio
ϕ_x	centerplane-normal temperature excess ratio
ϕ_z	spanwise-surface temperature excess ratio
ρ	density

Subscripts

0	at the vertical wall
∞	ambient value
s	maximum at the heat source

Superscript

+	dimensional value

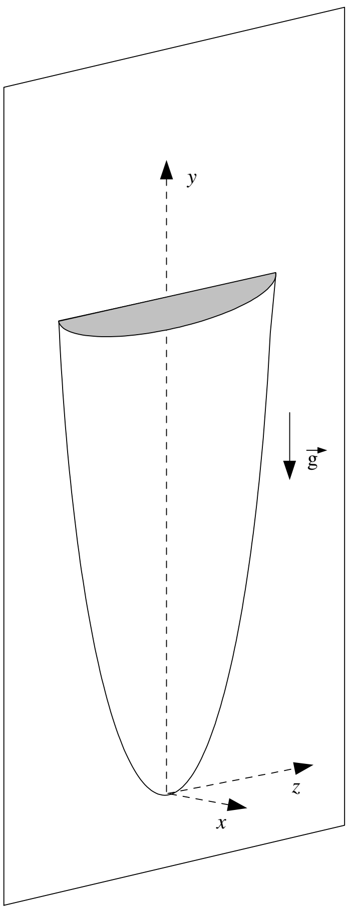

Fig. 1. General schematic for the studied concentrated heat source problem.

change with temperature in the buoyancy term. The finite volume control method was used for the discretization of the elliptic governing equations, the steady state three-dimensional versions of continuity, momentum and energy equations.

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

Momentum equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial P}{\partial x} + Pr\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + Pr\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + Ra_d^* Pr T \tag{3}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + Pr\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

The equations are made dimensionless using d characteristic length of the heat source, α thermal diffusivity, ρ density, and q'' the heat flux prescribed at the source as reference quantities. Additionally, the following dimensionless parameters have been defined by Carey and Mollendorf [10].

Surface temperature excess ratio above heat source

$$\phi = \frac{T_0 - T_\infty}{T_s - T_\infty} \tag{6}$$

Centerplane-normal temperature excess ratio

$$\phi_x = \frac{T(x,y,0) - T_\infty}{T_0 - T_\infty} \tag{7}$$

Spanwise-surface temperature excess ratio

$$\phi_z = \frac{T(x,0,z) - T_\infty}{T_0 - T_\infty} \tag{8}$$

where $T_0 = T(x,0,0)$ is the temperature at the vertical wall, T_∞ the ambient temperature and T_s the maximum temperature at the heat source.

Modified Grashof number

$$Gr_d^* = \frac{g\beta d^4 q''}{k\nu^2} \tag{9}$$

Modified Rayleigh number

$$Ra_d^* = Gr_d^* Pr \tag{10}$$

where g is the gravity acceleration, d the characteristic length of the heat source, k the thermal conductivity, q'' the heat flux prescribed at the source, β the volumetric coefficient of thermal expansion and ν the kinematic viscosity.

The computational domain was 1.5 m of height, 1.0 m of depth and 2.0 m of width, in which the left surface was assumed adiabatic, with no-slip condition, the others were assumed to be open boundaries. At the top boundary a condition of local developed flow was assumed (zero normal gradients). For the other surfaces zero normal gradients for velocities and ambient temperature were assumed. The boundary conditions were established to simulate a discrete heat source flush mounted on an adiabatic vertical wall.

2.2. Numerical procedure

The numerical solution was obtained using the SIMPLE algorithm, since the flow was assumed incompressible. The interpolation of gradients of velocities and temperatures used the QUICK algorithm.

Unstructured base grid was used in the region of largest temperature gradients, refined until no significant variations in the results were observed. Typical 2D and 3D grids used were of 1×10^4 and 2×10^5 order of number of cells. The resulting algebraic system was solved by the point-to-point Gauss–Seidel algorithm, which is known to have slow convergence, and to accelerate this convergence the algebraic multigrid method was used. The governing equations were solved in the dimensional form and then the results were arranged in the appropriated form. Fluid properties were taken from Bejan [12]. Validation of the numerical procedure was checked by solving the 2D and 3D problems presented by De Vahl Davis [13] and Fusegi et al. [14], respectively.

3. Results

The two-dimensional case is often used to model the flow due to a line heat source, as in Milanez and Bergles [3]. This case was solved using properly the boundary conditions mentioned above. The decaying laws for the temperature far downstream of a heat source date back to the pioneering work of Zeldovich [15]. Jaluria and Gebhart [8] used similarity solution to show that the wall temperature decays with y^n, where $n = -3/5$ and y is the dimensionless vertical distance above the heat source. In Fig. 2 the dimensionless temperature decay as a function of the dimensionless vertical distance above the heat source can be noticed. A good agreement was found between the results presented here and the scaling law derived from the similarity approach. The fitted curve was calculated using the points where $y > 3$.

The three-dimensional case was experimentally studied by Carey and Mollendorf [10]. They presented detailed measurements of the temperature above a discrete heat source on a plexiglass wall and concluded that the surface temperature decays proportional to $(y)^{-0.77}$, where d is the source characteristic length. This result was questioned by Higuera and Weidman [11] who carried out a scale analysis before solving the governing equations. They found that the surface temperature far downstream of the heat source decays proportional to y^{-1}, as a point source plume. Fig. 3 shows a good agreement of the numerical results with that reported in [11].

Higuera and Weidman argued that the measurements carried out by Carey and Mollendorf were taken only at

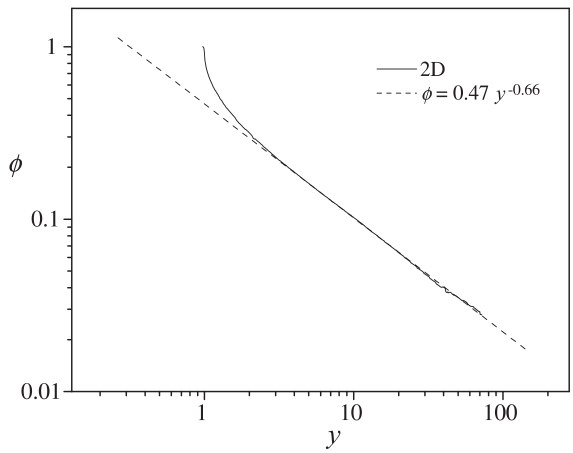

Fig. 2. Comparison between analytical and numerical results (2D).

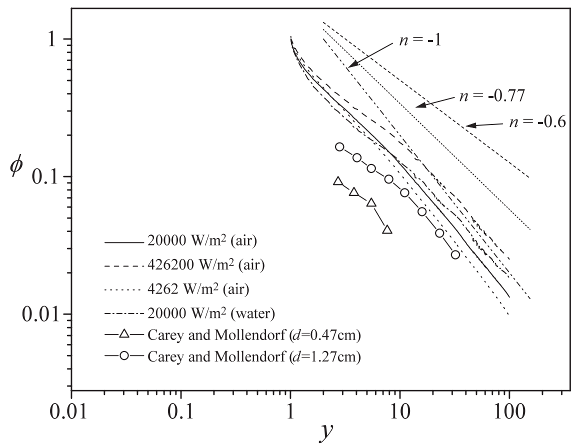

Fig. 3. Comparison between analytical and numerical results (3D).

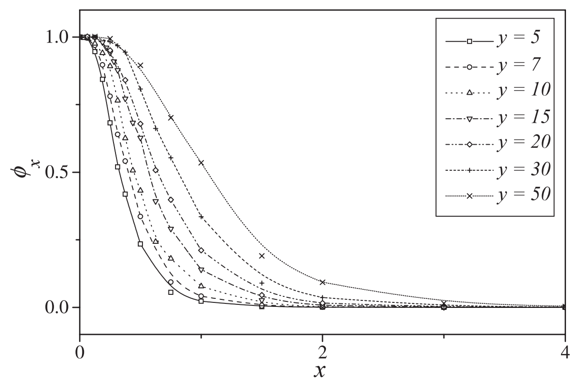

Fig. 4. Centerplane temperature with surface-normal coordinate.

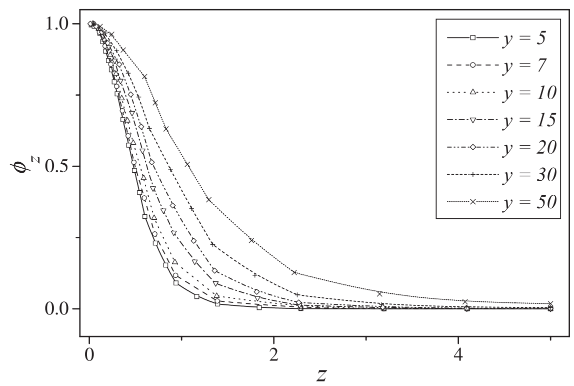

Fig. 5. Spanwise-surface temperature with spanwise coordinate.

moderate distances downstream of the source, where apparently, according to them, the flow was intermediate between a two- and three-dimensional plume, as can be seen in Fig. 3. Additionally, this transition from two- to three-dimensional plume should be dependent of the source dimension, because there would be no transition at all if the source were infinitesimal. Carey and Mollendorf used two different source sizes, a 1.27 cm square and other circular with diameter of 0.47 cm. Nevertheless, the results they presented do not allow any conclusion about the dependence of the transition with the source size, although, in fact, it can be seen that from $y \simeq 8$, for the smaller source, and from $y \simeq 10$, for the other one, the experimental results approach the theoretical predictions.

To study the behavior of the thermal boundary layer, the temperature profile downstream the heat source in the planes normal (xy) and tangent (zy) to the wall is analyzed. Two dimensionless parameters that represent the temperature difference between the wall and the ambient temperature were defined as in [10], see Eqs. (7) and (8).

Carey and Mollendorf measured the temperature in these planes along the coordinate y in two points along (xy) and (zy), namely $y = 7.0$ and 10.2 cm, corresponding to $y = 5.5$ and 8.0. A different similarity scale was determined for the temperature profiles in the two planes considered (x/y in plane xy and $z/y^{1/5}$ in plane zy), stressing the three-dimensional structure of the flow.

Figs. 4 and 5 present the dimensionless temperature profiles taken at several positions in planes xy and zy, including the region studied in Ref. [10], $y < 10$. It can be noticed that the thermal boundary layer thickness in plane zy is about 25% higher than that of plane xy, even for points sufficiently far from the heat source, that is, in the three-dimensional region fully developed. In order to determine the similarity scale for each plane studied, curves of x and z as functions of y, parametrized by ϕ_x and ϕ_z, were plotted in log–log graphs to obtain coefficients m_x and m_z for relations $x/y^{m_x} = C$ and $z/y^{m_z} = C$, where C is an arbitrary constant. Figs. 6 and 7 exhibit average curves and the respective coefficients numerically obtained and they are different from those obtained by Carey and Mollendorf.

For the xy plane the value of the coefficient numerically obtained was 0.49 ± 0.01, which is different from the experimental value of 1.0. According to the study of Higuera and Weidman [11], the theoretical value of the similarity scale for x and z are identical and equal to 0.5, therefore the numerical result seems to represent the theoretical value better than the experimental. This discrepancy between the experimental and the theoretical value is probably due to the region where the measurements were taken. As mentioned previously, the flow can be considered fully developed only after $y \simeq 10$.

For the zy plane, the curve exhibits an evident change of behavior at $y = 10$, stressing that is in this region that

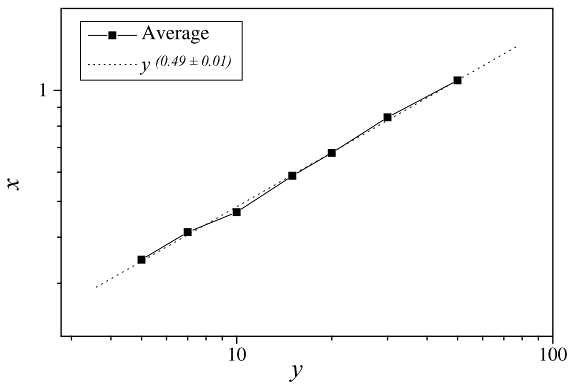

Fig. 6. Normalization factor for the centerplane temperature variation.

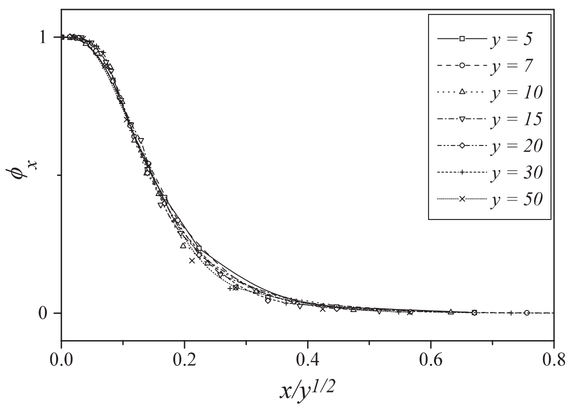

Fig. 8. Centerplane temperature with normalized surface-normal coordinate.

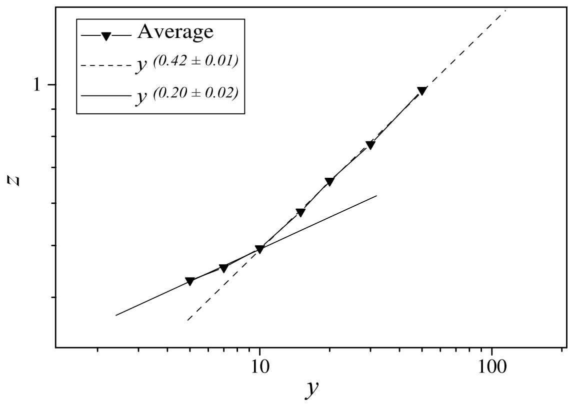

Fig. 7. Normalization factor for the spanwise-surface temperature variation.

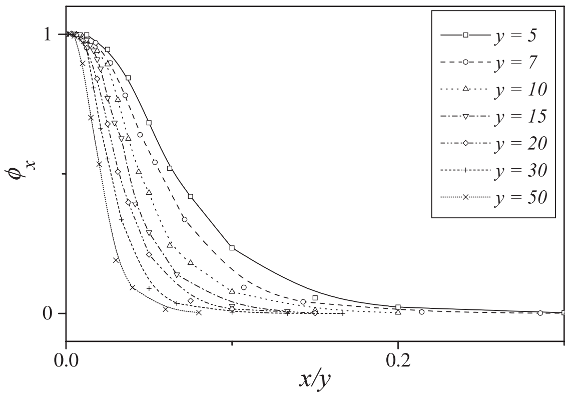

Fig. 9. Centerplane temperature with normalized surface-normal coordinate.

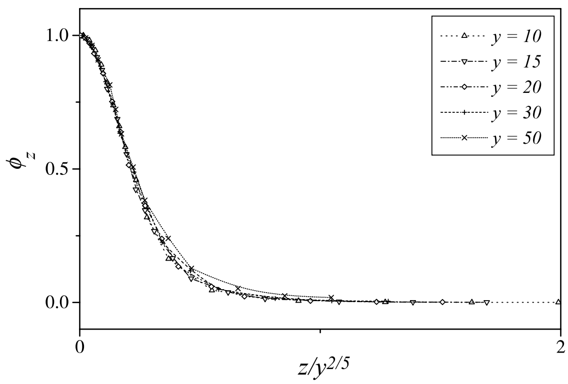

Fig. 10. Spanwise-surface temperature with normalized spanwise coordinate.

the flow transition is taking place. For $y < 10$, the coefficient obtained numerically (0.20 ± 0.02) coincides with the experimental value (1/5). However, for $y > 10$, the numerical value is 0.42 ± 0.01, exhibiting a more pronounced decay closer to the theoretical value of 0.5. This theoretical value can be questioned, because the model assumes identical behaviors for planes *xy* and *zy*, that is, the same behavior expected for an axisymmetric plume, therefore ignoring the presence of the drag force at the wall.

Figs. 8 and 9 show the numerical results normalized by the coefficient obtained numerically (Fig. 8) and experimentally (Fig. 9) [10]. It may be observed that the numerical coefficient is a better representation for a similarity scale.

Figs. 10 and 11 depict the numerical data respective to plane *zy*, normalized by the coefficient obtained numerically (Fig. 10) and experimentally (Fig. 11) [10]. It can be noticed that the numerical coefficient is a better representation for a similarity scale, while the experimental fits well only for the region $y < 10$.

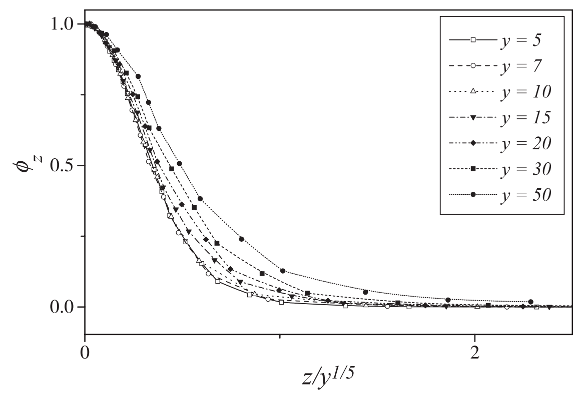

Fig. 11. Spanwise-surface temperature with normalized spanwise coordinate.

4. Conclusions

The numerical results presented confirm in part the theoretical behavior established in [11] and show the existence of a transition between the two- and three-dimensional plume. Additionally, similarity scales for the thermal boundary layer were proposed for the *xy* and *zy* planes. The influence of the source size on the transition region deserves more attention. The presence of this region of transition emphasizes the complexity of this kind of flow, where two- or three-dimensional effects must not be studied separately, mainly on electronic packaging where heat sources are in that transition region.

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